

THE DIFFERENTIAL PERTURBATIVE METHOD APPLIED TO SENSITIVITY ANALYSIS FOR WATERHAMMER PROBLEMS IN HYDRAULIC NETWORKS

(This paper was published in Applied Mathematical Modelling 25 (2001) 1117-1138.
See in <http://www.elsevier.com/locate/apm>)

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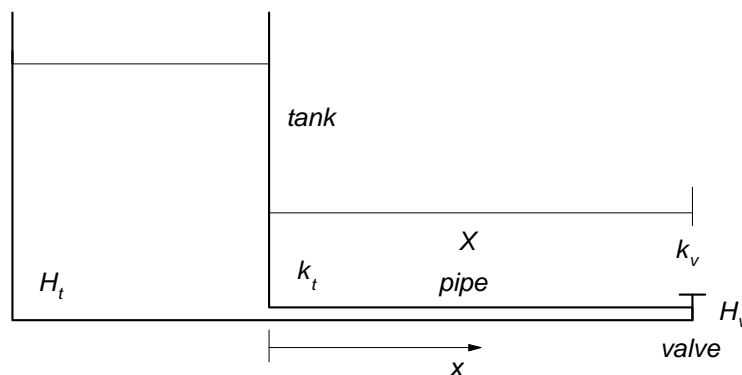
ABSTRACT

In this paper the differential perturbative method was applied to the sensitivity analysis for waterhammer problems in hydraulic networks. Starting from the classical waterhammer equations in a single-phase liquid with friction (the direct problem) the state vector comprising the piezometric head and the velocity was defined. Applying the differential method the adjoint operator, the adjoint equations with the general form of their boundary conditions, and the general form of the bilinear concomitant were calculated for a single pipe. The calculation of the sensitivity coefficients takes into account the cases in which the parameters under consideration influence the initial condition. For these cases, the calculation can be performed by solving sequentially two perturbative problems: the first one is non-steady, while the second one is steady, with an appropriate selection of a

weight function coming from the unsteady perturbative problem. As an example, a constant-level tank connected through a pipe to a valve discharging to atmosphere was considered. The corresponding sensitivity coefficients due to the variation of different parameters by using both the differential method and the response surface generated by solver of the direct problem, were also calculated. The results obtained with these methods show excellent agreement. In this summary single part of an application example is shown.

APPLICATION EXAMPLE

Let us consider the problem of a single pipe connected at the end $x=0$ to a constant-level tank, while the end $x=X$ is connected to a valve discharging to atmosphere, as shown then:



Direct equations. The direct boundary conditions for this case are:

$$C_1 \equiv H + \frac{1}{2g} (V^2 + k_t V |V|) - H_t = 0 \quad \text{at } x = 0 \quad (1)$$

$$C_2 \equiv H - \frac{1}{2g} k_v V |V| - H_v = 0 \quad \text{at } x = X \quad (2)$$

$$C_3 \equiv H - \tilde{H} = 0 \quad \text{at } t = 0 \quad (3)$$

$$C_4 \equiv V - \tilde{V} = 0 \quad \text{at } t = 0 \quad (4)$$

were C_1 and C_2 represent the boundary conditions at the tank and at the valve, while C_3 and C_4 represent the initial conditions in steady state.

Derived equations. The derived boundary conditions result:

$$H_{/i} + V_{/i} \frac{1}{g} (V + k_t |V|) + C_{1/i} = 0 \quad \text{at } x = 0 \quad (5)$$

$$H_{/i} - V_{/i} \frac{k_v}{g} |V| + C_{2/i} = 0 \quad \text{at } x = X \quad (6)$$

$$H_{/i} - \tilde{H}_{/i} = 0 \quad \text{at } t = 0 \quad (7)$$

$$V_{/i} - \tilde{V}_{/i} = 0 \quad \text{at } t = 0 \quad (8)$$

Adjoint equations and bilinear concomitant. The adjoint boundary conditions result:

$$C_1^* \equiv \frac{a^2}{g} H^* - (V + k_t |V|) V^* = 0 \quad \text{at } x = 0 \quad (9)$$

$$C_2^* \equiv \frac{a^2}{g} H^* + k_v |V| V^* = 0 \quad \text{at } x = X \quad (10)$$

$$C_3^* \equiv H^* = 0 \quad \text{at } t = 0 \quad (11)$$

$$C_4^* \equiv V^* = 0 \quad \text{at } t = 0 \quad (12)$$

The bilinear concomitant can be calculated as:

$$\begin{aligned} P = & \int_0^X [H^*(x,0) \tilde{H}_{/i}(x) + V^*(x,0) \tilde{V}_{/i}(x)] dx \\ & + \int_0^T C_{2/i}(X,t) g V^*(X,t) dt \\ & - \int_0^T C_{1/i}(0,t) g V^*(0,t) dt \end{aligned} \quad (13)$$

Definition of the sensitivity problem. We consider the hydraulic system in a steady state for $t \leq 0$. For $t \geq 0$ the valve is operated in such a way that the friction coefficient changes linearly from k_{vi} to k_{vf} in a time interval τ , as shown then:

$$k_v = \begin{cases} k_{vi} + \frac{k_{vf} - k_{vi}}{\tau} & \text{for } 0 \leq t \leq \tau \\ k_{vf} & \text{for } t > \tau \end{cases} \quad (14)$$

Two different values of the reference time T ($T = \tau/2$ and $T = 20\tau$) were chosen. Consequently. The closure time τ has been chosen equal to the period of the perturbation ($4X/a$). For $T = \tau/2$ we get the maximum value of the piezometric head at the valve, while for $T = 20\tau$ a new steady state is achieved. They are tabulated in Tables 1 and 2 along with the sensitivities evaluated by obtaining direct solutions with the selected perturbed parameters, using the code WHAT.

Since the parameters chosen are only related to the boundary condition at the valve, namely Eq. (117), we finally get for the sensitivity coefficients:

$$\frac{\delta R}{\delta k_{vf}} = \int_0^T \frac{\partial C_2}{\partial k_{vf}} g V^*(X, t) dt \quad (15)$$

where:

$$\frac{\partial C_2}{\partial k_{vf}} = \begin{cases} 0 & \text{for } t < 0 \\ -\frac{1}{2g} V |V| \frac{t}{\tau} & \text{for } 0 \leq t \leq \tau \\ -\frac{1}{2g} V |V| & \text{for } t > \tau \end{cases} \quad \frac{\delta R}{\delta \tau} = \int_0^T \frac{\partial C_2}{\partial \tau} g V^*(X, t) dt$$

$$\frac{\partial C_2}{\partial \tau} = \begin{cases} 0 & \text{for } t < 0 \\ \frac{1}{2g} \frac{k_{vf} - k_{vi}}{\tau} V |V| \frac{t}{\tau} & \text{for } 0 \leq t \leq \tau \\ 0 & \text{for } t > \tau \end{cases}$$

Table 1. Sensitivity coefficients for $T = \tau/2$.

<i>Node</i>	<i>01</i>		<i>06</i>		<i>11</i>	
	WHAT	SANWHAT	WHAT	SANWHAT	WHAT	SANWHAT
$\frac{\delta H}{\delta k_{vf}} [m]$	2,83 10^{-5}	2,83 10^{-5}	9,65 10^{-3}	9,64 10^{-3}	2,06 10^{-2}	2,06 10^{-2}
$\frac{\delta V}{\delta k_{vf}} [m/s]$	-1,81 10^{-4}	-1,81 10^{-4}	-1,71 10^{-4}	-1,71 10^{-4}	-1,38 10^{-4}	-1,38 10^{-4}
$\frac{\delta H}{\delta \tau} [m]$	-0,34	-0,33	-115,6	-115,5	-247,0	-247,1
$\frac{\delta V}{\delta \tau} [m/s]$	2.170	2.171	2.050	2.051	0.65	0.66

Table 2. Sensitivity coefficients for $T = 20 \tau$.

Node	01		06		11	
	WHAT	SANWHAT	WHAT	SANWHAT	WHAT	SANWHAT
$\frac{\delta H}{\delta k_{vf}} [m]$	4,11 10^{-6}	4,14 10^{-6}	6,15 10^{-5}	5,27 10^{-5}	1,19 10^{-4}	1,01 10^{-4}
$\frac{\delta V}{\delta k_{vf}} [m/s]$	-1,13 10^{-4}	-1,14 10^{-4}	-1,13 10^{-4}	-1,14 10^{-4}	-1,13 10^{-4}	-1,14 10^{-4}
$\frac{\delta H}{\delta \tau} [m]$	-6,0 10^{-8}	-7,1 10^{-8}	-8,2 10^{-5}	-8,4 10^{-5}	-8,2 10^{-5}	-8,7 10^{-5}
$\frac{\delta V}{\delta \tau} [m/s]$	1,8 10^{-6}	2,0 10^{-6}	1,8 10^{-6}	2,0 10^{-6}	3,3 10^{-6}	3,6 10^{-6}

CONCLUSIONS

From Tables 1 and 2 it can be observed that the agreement between the results obtained from the codes WHAT and SANWHAT is excellent for short observation times.

The development of the sensitivity theory by the differential method for a general waterhammer problem was outlined. The adjoint equations and the general form of the bilinear concomitant were obtained. The methodology was applied to a simple problem, showing excellent agreement between the sensitivity coefficients calculated with the differential method and the ones obtained via the solution of many perturbed direct problems. The authors hope that this paper will encourage the use of perturbative methods for sensitivity analysis in different areas of Engineering Science.

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