

# MONTE-CARLO SIMULATION FOR FRAGMENT MASS AND KINETIC ENERGY DISTRIBUTIONS FROM NEUTRON INDUCED FISSION OF $^{235}\text{U}$

Montoya M. <sup>(1),(2)</sup> [mmontoya@ipen.gob.pe](mailto:mmontoya@ipen.gob.pe); Saettone E. <sup>(2)</sup>; Rojas J. <sup>(1),(3)</sup> [vrojas@ipen.gob.pe](mailto:vrojas@ipen.gob.pe)

(1) Instituto Peruano de Energía Nuclear / Lima, Perú

(2) Facultad de Ciencias, Universidad Nacional de Ingeniería / Lima, Perú

(3) Facultad de Ciencias Físicas, Universidad Nacional Mayor de San Marcos / Lima, Perú

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## ABSTRACT

Mass and kinetic energy distribution of nuclear fragments after neutron induced fission of  $^{235}\text{U}$  have been studied using a Monte-Carlo simulation. Besides that the pronounced peak in the standard deviation of the kinetic energy  $\sigma_E(m)$  at the mass number around  $m = 110$  was reproduced, a second peak was found at  $m = 126$ . These results are in good agreement with experimental data obtained by Belhafaf *et al.* We have concluded that the obtained results are consequence of the characteristics of neutron evaporation for the fragments and sharp variation on primary mass yield curve.

## 1. INTRODUCTION

Since the discovery of the neutron induced fission of uranium by Hahn and Strassmann in 1938 [1], a big effort to measure the fission parameters and to understand the involved process was spent. Nowadays several aspects of heavy nuclei fission seem to be clarified. Meitner and Frisch suggested a theoretical explanation based on a nuclear liquid-drop model [2], and in a recent paper [3] this model was revisited and pointed out the nuclear surface-curvature terms and their effects.

It is known that the de-excitation by fission of heavy nuclei depends of the quantum properties of the saddle point and the associated fission barrier. The detection of fission isomers have been interpreted by the secondary well in the fission barrier [4]. The nascent fragments begin to be formed at the saddle point and the system fall down to the fission valley (energetically preferred paths to fission) and end at the scission configuration where fragments interact only by Coulomb force. At scission, fragments had acquired a pre-scission kinetic energy. Over

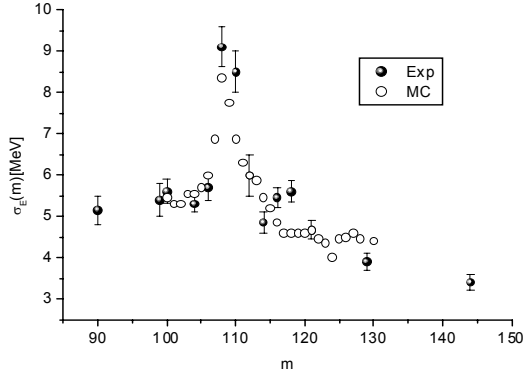
the fission valley, the system could be described by collective variables (deformation, vibration, rotation, etc.) and intrinsic variables (quasi-particles excitations). Nevertheless, the dynamic of fission process is not completely understood. An open question is the coupling between collective and intrinsic degrees of the freedom during the descend from the saddle to scission.

In the low-energy fission, several final fragment characteristics can be explained in terms of a static scission model of two coaxial juxtaposed deformed spheroidal fragments, provided shell effects, affecting the deformation energy of the fragments. These shell effects corrections, determined by Strutinsky prescription and discussed by Dickmann *et al.* [5] and by Wilkins [6], subsequently generate secondary minima in the total potential energy surface of fragments having some particular neutron or proton shell configurations. If the final fragment characteristics were governed by the properties of the fragments themselves, a basic argument in any statistical theory, one would expect an increase in the width of the kinetic energy distribution curve for fragment masses,  $A$ , having the above mentioned special neutron or proton shell arrangements.

In order to search an answer to this question the most studied fission parameters are the mass  $F(A)$  and kinetic energy  $E_K(A)$  distribution of primary (pre-neutron emission) fragments. Nevertheless, measurement can be carried out only on final fragments (post neutron emission) mass ( $m$ ) and kinetic energy  $E_K(m)$  distribution. Therefore it is crucial to find out the relation between  $E_K(m)$  and  $E_K(A)$  distributions.

For neutron induced fission of  $^{235}\text{U}$ , the  $E_K(m)$  distribution was measured by Brissot *et al.* [7]. This distribution was represented by the mean

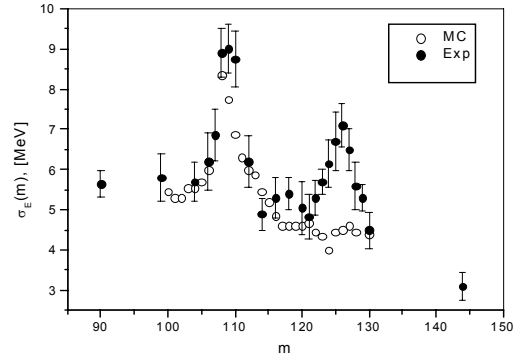
value of the kinetic energy  $\overline{E_K}$  and the standard deviation of kinetic energy  $\sigma_{EK}(m)$ , as function of the final mass  $m$ . Fig. 1, shows the one pronounced peak of  $\sigma_{EK}(m)$  at  $m = 110$ , and Monte-Carlo simulation of  $\sigma_{EK}(m)$  from a primary distribution without peaks. This result suggests that the peak does not exist at the  $\sigma_{EK}(m)$  of primary fragment kinetic energy as function of the primary fragment mass.



**Figure 1.** Experimental (full circles) and simulated by Monte-Carlo (open circles) standard deviation of final fragment kinetic energy as function of the final mass [7].

In a latter experiment, Belhafaf *et al.* [8] repeated the experiment of Brissot *et al.* for neutron induced fission of  $^{235}\text{U}$ , obtaining also another peak around  $m = 126$  (see Fig. 2). A Monte-Carlo simulation made by these authors, from a distribution without a peak, did not reproduce the experimental on  $\sigma_{EK}$  at  $m = 126$ , only the peak at 110. They suggested that this peak must exist in the primary fragment distribution. They have fitted the experimental data from a distribution with a peak at  $m = 126$ .

In this paper, we present a new Monte-Carlo simulation results for low energy fission of  $^{235}\text{U}$ . Both mass and kinetic energy distributions on the primary and the final fission fragments were numerically calculated. It is shown that both peaks at  $m = 110$  and  $126$  on  $\sigma_{EK}$ , as function of the final mass  $m$ , were reproduced without assuming certain initial structure.



**Figure 2.** Experimental (full circles) and simulated by Monte-Carlo (open circles) standard deviation of final fragment kinetic energy as a function of the final mass [8].

## 2. MONTE-CARLO SIMULATION MODEL

We assume that total kinetic energy distribution of fission fragments is approximated to the normalized Gauss distribution [11]

$$P(E) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(E - E_0)^2}{2\sigma^2}\right] \quad (1)$$

where  $E$  and  $E_0$  are the total kinetic energy and most probable total kinetic energy, respectively, and  $\sigma$  is the variance of total kinetic energy.

It is known that for thermal neutron induced fission of  $^{235}\text{U}$  nuclei preferentially splits asymmetrically in two fission fragments [12,13]. Then we have a heavy  $A_H$  and light  $A_L$  fragments, so that  $A_L + A_H = A_0 = 236$ .

Let  $\overline{E_{KT}}(A)$  and  $\sigma_{EKT}(A)$  be the mean of initial total kinetic energy and the standard deviation of kinetic energy distribution as a function of fragment mass  $A$ . These distributions without peaks are assume as input data for simulation. Let  $\langle \nu \rangle$  be the average number of neutrons emitted by fragments.

The total number of emitted neutrons will be a function of the excitation energy  $U$ .

$$U = Q - E_{KT} \quad (2)$$

where  $Q$  is the available energy for fission and  $E_{KT}$  is the initial total fragments kinetic energy. In order to simulate the mass and kinetic energy distribution of the fragments for each fission event, the total kinetic energy  $E_{KT}$  is hosen

randomly from a Gaussian distribution with a mean value in  $\overline{E_{KT}}(A)$  and standard deviation  $\sigma_{EKT}(A)$

The primary fragment kinetic energy are given as,

$$E_{KP}(A) = E_{KT}(A) \frac{A}{A_0}, \quad (3)$$

We assume that the neutron emission by fragments take place according to the expression,

$$\nu(A) = \langle \nu(A) \rangle \left\{ 1 + \frac{E_{KT}(A) - \overline{E_{KT}}(A)}{3\sigma_{KET}(A)} \right\} \quad (4)$$

The final mass of the fragment is,

$$m = A - \nu \quad (5)$$

We assume that the lost of energy by fragments take place only due to the neutron evaporation but not by gamma radiation or other processes. Thus, if we neglect recoil effects of neutron emission the final kinetic energy is given as

$$E_{KF} = E_{KP} \left(1 - \frac{\nu}{A}\right) \quad (6)$$

The total number of fission events of  $^{236}\text{U}$  for an acceptable statistic during the simulation was of the order of  $10^8$ . The random numbers with required normal distribution were generated using the method of Box-Muller.

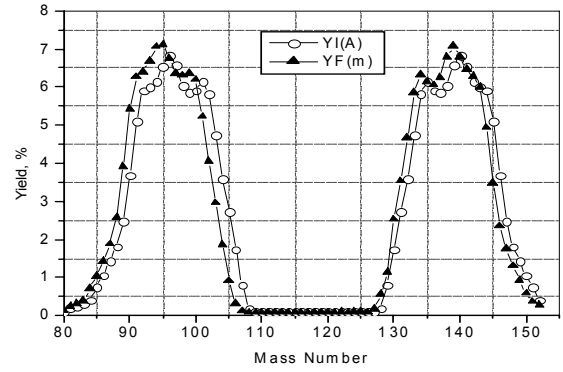
The standard deviation of interested quantities, like the kinetic energies of final fragments given by the eq. (6) were calculated as,

$$\sigma_{EKF}^2(m) = \frac{\sum_{j=1}^{N_j} E_{KFj}^2(m)}{N_j(m)} - \langle E_{KF}(m) \rangle^2 \quad (7)$$

where  $\langle E_{KF}(m) \rangle$  is the most probable final kinetic energy of fragment with mass  $m$ , and  $N_j(m)$  is the number of fission for a given mass.

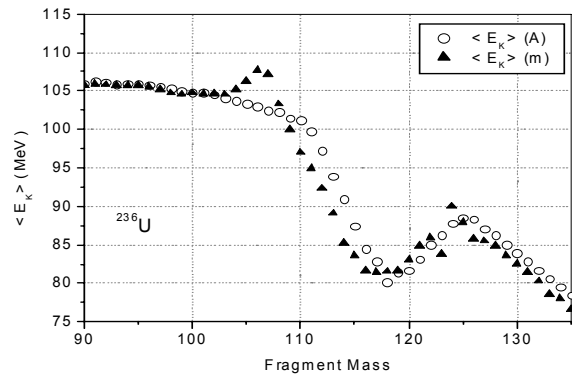
### 3. RESULTS OF SIMULATIONS AND DISCUSSION

The simulated final mass yield curve ( $m$ ) and the primary mass yield curve  $Y(A)$  is presented in Fig. 3. The  $Y(m)$  is shifted from  $Y(A)$ , due to the neutron emission



**Figure 3.** Monte-Carlo simulation results for initial (o) and final (▲) mass yields from neutron induced fission of  $^{235}\text{U}$ .

The total primary kinetic energy, generated as mentioned above in sect.2, have a Gaussian distribution. Both the primary and the final fragment kinetic energy distribution were derived from the total kinetic energy distribution using the eq.(3) and (6). Fig. 4 shows the simulated mean fragment kinetic energy, for primary (o) and final (▲) fragments as a function of the fragment mass.

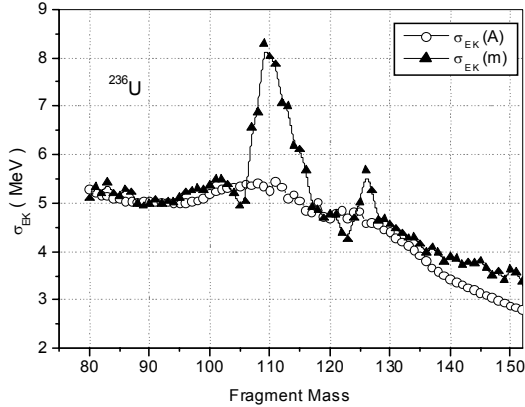


**Figure 4.** Average kinetic energy of initial (o) and final (▲) fragments for neutron induced fission of  $^{235}\text{U}$ .

Notice that the difference of both curves around the fragments mass at 110 and at 125. For the symmetrical fission fragments ( $A=118$ ) the kinetic energy present minimum with an approximately 81 MeV.

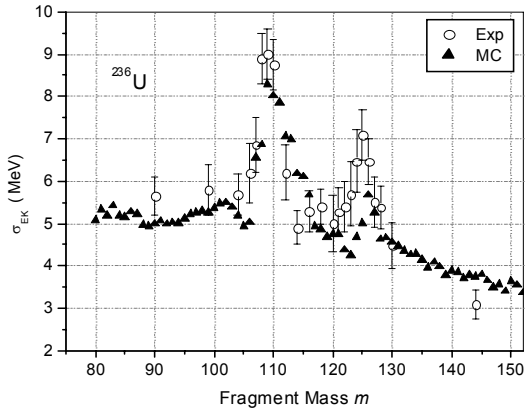
Fig. 5 shows both primary and final fragment kinetic energy standard deviation from its mean

value as a function of the fragment mass. One relevant feature is the appearance of a pronounced peak at  $m = 110$ , and another small peak around  $m = 126$ . It is pointed out that in the simulation of the primary fragment kinetic energy distribution (see fig.5, open circles), has no peaks over the fragment mass between 80 and 150.



**Figure 5.** Simulation results of the standard deviation of kinetic energy for final ( $\blacktriangle$ ) and primary ( $\circ$ ) fragment mass for neutron induced fission of  $^{235}\text{U}$ .

In the Fig.6, the experimental data and the present Monte-Carlo ( $\blacktriangle$ ) simulation results for fragment kinetic energy standard deviation were plotted. The simulated results fitted well the experimental data.



**Figure 6.** Standard deviation of fragment kinetic energy distribution as function of the fragment mass: ( $\blacktriangle$ ) - present MC results, ( $\circ$ ) experimental data [8].

The initial total kinetic energy used, and the simulated primary fragments kinetic energies has no peaks. The behavior of the standard deviation of final kinetic energy distribution is not being caused by the structure on the primary kinetic energy distribution. The presence of peaks could be associated with neutron emission peculiarities. Average neutron number of emitted

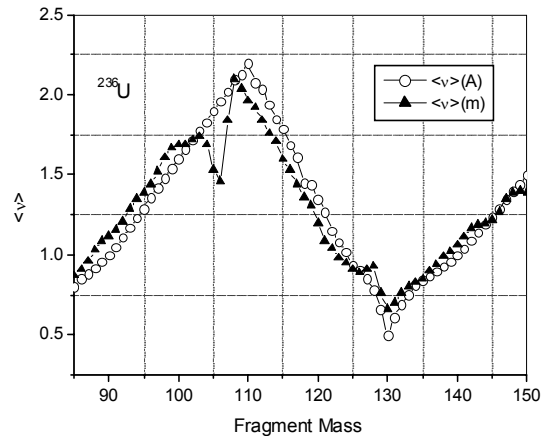
by fragments as a function of the primary fragment mass  $A$  and as a function of the final fragment mass  $m$ , respectively, are presented in Fig. 7. The neutron emission, produced a peculiar structure on  $\nu(m)$  curve, that is not shown on primary data for  $\nu(m)$ , are produced.

### 3.1 INTERPRETATION FOF NEUTRON EMISSION EFFECTS ON FINAL MASS AND KINETIC ENERGY DISTRIBUTION

In the Monte-Carlo simulation of an experiment at low energy fission, we assume that there is no recoil by neutron emission from the fragment, which then loose kinetic energy by just losing mass.

For light fragment this shift is significant, because the width of  $E_{KF}$  distribution may be hence enlarge approximately by 1 MeV. To better understand this effect, let us assume that the neighborhood of  $m = 100$ , the average  $E_K$  is 100 MeV, and the standard deviation  $\sigma_{EK}$  is 6 MeV. Let us also assume that fragments in the higher part of  $E_K$  distribution ( $E_K > 100$  MeV) emit 0 neutron, and fragments in the lower part of  $E_K$  distribution ( $E_K < 100$  MeV) emit 1 neutron. If the

yield and  $\overline{E}_K(A)$  curves are flat, i.e., for each value of  $m$ , half of fragments corresponds to  $A_L = m_L + 1$ , the  $E_{KF}$  distribution will be approximately 1 MeV larger than  $E_{KP}$  distribution.



**Figure 7.** The average number of emitted neutrons from neutron induced fission of  $^{235}\text{U}$ : ( $\circ$ ) as function of primary fragment mass  $m$ , ( $\blacktriangle$ ) - as function of final fragment mass  $A$ .

The  $E_{KF}$  distribution width is very dependent of the mass yield curve. In the above case, if there is a sharp increasing of yield for  $m$  to  $m-1$ , the

width of  $E_{KF}$  distribution is depleted approximately to the half. For  $m = 100$ , only the higher half of the  $E_{KP}$  distribution ( $E > 100$  MeV) will be taken into account. This is because for this half of  $E_{KP}$  distribution there is no neutron emission. If the yield curve continues to increase, the width will be also depleted.

When the Y curve returns to be flat, the  $\sigma_{EKF}$  will return to be similar to  $\sigma_{EKP}$  around the inflection point of yield mass. If the Y curve fall down with diminishing of A,  $\sigma_{EKF}$  will fall again, producing a peak on this curve.

#### 4. CONCLUSION

In this work, on a simple model for the neutron emission by fragments a Monte-Carlo simulation for mass and kinetic energy distribution of thermal neutron induced fission fragments has been considered. Final fragments -post neutron emission- have eroded kinetic and mass values in comparison with initial -pre neutron emission- fragment kinetic energy and mass values. This fact, combined with sharp variation on primary mass yield curve as a function of primary mass, produce structures on final values distributions. In the case of neutron induced fission of  $^{235}\text{U}$ , peaks at  $m = 110$  and  $m = 126$  on  $\sigma_{EKF}$  are produced as an effect of neutron emission by fission fragments.

Since experimental investigations do not provide any information on the initial energy distribution of single fission fragments and due to crucial effect of neutron emission on final fragments mass and kinetic energy distribution, Monte-Carlo simulation is useful in order to relate primary and final fragments distributions.

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