

# The most compact scission configuration of fragments from low energy fission of $^{234}\text{U}$ and $^{236}\text{U}$

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## Abstract

Using a time of flight technique, the maximal values of kinetic energy as a function of primary mass of fragments from low energy fission of  $^{234}\text{U}$  and  $^{236}\text{U}$  were measured by Signarbieux *et al.* From calculations of scission configurations, it is concluded that, for those two fissioning systems, the maximal value of total kinetic energy corresponding to fragmentations ( $_{42}\text{Mo}_{62}$ ,  $_{50}\text{Sn}_{80}$ ) and ( $_{42}\text{Mo}_{64}$ ,  $_{50}\text{Sn}_{80}$ ) respectively, are equal to the available energies, and their scission configurations are composed by a spherical heavy fragment and a prolate light fragment, both in their ground state.

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Keywords: Low energy fission;  $^{234}\text{U}$ ;  $^{236}\text{U}$ ; fragment kinetic energy; cold fission

## Resumen

Usando una técnica de tiempo de vuelo, Signarbieux *et al.* midieron el valor máximo de la energía cinética total en función de la masa primaria de los fragmentos de la fisión de baja energía de  $^{234}\text{U}$  y  $^{236}\text{U}$ . De los cálculos de las configuraciones de escisión, puede concluirse que, para esos dos sistemas físiles, el valor máximo de la energía cinética corresponde a las fragmentaciones ( $_{42}\text{Mo}_{62}$ ,  $_{50}\text{Sn}_{80}$ ) y ( $_{42}\text{Mo}_{64}$ ,  $_{50}\text{Sn}_{80}$ ), respectivamente, son iguales a los valores disponibles de energía; y sus configuraciones de escisión están compuestas por un fragmento pesado esférico y un fragmento liviano prolato, ambos en sus estados fundamentales.

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Palabras clave: Fisión a baja energía;  $^{234}\text{U}$ ;  $^{236}\text{U}$ ; energía cinética de fragmentos; fisión fría

## 1. Introduction

One of the most studied quantities to understand the fission process is the fragment mass and kinetic energy distribution, which is very closely related to the topological features in the multi-dimensional potential energy surface [1]. Structures on the distribution of mass and kinetic energy may be interpreted by shell effects on potential energy of the fissioning system, determined by the Strutinsky prescription and discussed by Dickmann [2] and Wilkins [3].

In order to investigate the fragments with very low excitation energy, using the time of flight method, Signarbieux *et al.* [4] measured the fragment mass distribution for high values of fragment kinetic energy. Because in that kinetic energy region there is no neutron emission, the time of flight technique permits separate neighboring fragment masses. In this

work one calculates the deformations of those fragments which must correspond to the most compact scission configurations, i.e. to the highest values of Coulomb interaction energy between the two fragments.

## 2. The most compact scission configurations

In the process of thermal neutron induced fission of  $^{233}\text{U}$ , a composed nucleus  $^{234}\text{U}$  with excitation energy equal to neutron separation energy ( $B_n$ ) is formed first. Then, this nucleus splits in two complementary light and heavy fragments having  $A_L$  and  $A_H$  as mass numbers, and  $E_L$  and  $E_H$  as kinetic energies, respectively.

The Q-value of this reaction is given by the relation:

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$$Q = M(92,234) - M(Z_L, A_L) - M(Z_H, A_L) \quad (1)$$

where  $M(Z, A)$  is the mass of nucleus with  $Z$  and  $A$  as proton number and mass number, respectively.

The balance energy at scission configuration will be

$$Q + B_n = TKE_0 + CE + TDE + DXE; \quad (2)$$

where  $TKE_0$  is the pre-scission total kinetic energy;  $CE$  is the Coulomb interaction energy between fragments;

$$TDE = DE_L + DE_H, \quad (3)$$

is the total deformation energy, where  $DE_L$  and  $DE_H$  are the light and heavy fragment deformation energy, respectively; and

$$TXE = XE_L + XE_H, \quad (4)$$

is the total intrinsic excitation energy, where  $XE_L$  and  $XE_H$  are the light and heavy fragment intrinsic excitation energy, respectively.

If there is no neutron emission, the light and heavy fragments reach the detectors with their primary kinetic energies equal to  $KE_L$  and  $KE_H$ , respectively. The total primary fragments kinetic energy will be

$$TKE = KE_L + KE_H = TKE_0 + CE = Q + B_n - TDE - TXE \quad (5)$$

The maximal value of total kinetic energy is reached when the sum of  $TDE$  and  $TDX$  is minimal, i.e.

$$TKE_{\max} = (TKE_0 + CE)_{\max} = Q + B_n - (TDE - TXE)_{\min}. \quad (6)$$

The most compact scission configuration occurs when maximal value of coulomb energy is equal to the available energy, i.e.

$$CE_{\max} = Q + B_n. \quad (7)$$

In this case, from Eq. 5 one obtains the relations

$$TKE_{\max} = CE_{\max} = Q + B_n \quad (8)$$

and

$$DE_{\min} = 0, \quad DX_{\min} = 0 \text{ and } TKE_0 = 0. \quad (9)$$

Not always this situation is possible to occur. Nevertheless we can assume that for each mass fragmentation the maximal value of total kinetic energy is obtained for similar condition, i.e.  $TKE_0 = 0$ ,  $TXE = 0$  and  $TDE = TDE_{\min}$ .

### 3. Deformation energy

A fragment total energy ( $U$ ), composed by nucleons, is calculated using the Strutinsky method [5]. Strutinsky proposes to define a smooth function ( $\tilde{U}$ ), without fluctuation due to shell effects. The shell correction will be

$$\delta U = U - \tilde{U}$$

As a first approximation, we calculate the total energy by a liquid drop model type, using the mass formula of Myers and Swiatecky [6]. The shell correction ( $\delta U_Z$  and  $\delta U_N$ , corresponding to proton and neutron numbers, respectively) is calculated by the Strutinsky's method [5], using Nilsson Hamiltonian for harmonic axial symmetrical well, with spin-orbit and centrifuge corrections [7]:

$$V_{corr} = -\kappa \hbar \omega_0 \left\{ \hat{l} \cdot \hat{s} + \mu \left( \hat{l}^2 - \langle l^2 \rangle_N \right) \right\} \quad (10)$$

where  $\hat{l}$  is the orbital angular momentum operator,  $\hat{s}$  is the spin operator,  $\kappa$  and  $\mu$  are the Nilsson's constants. The constant of the harmonic oscillator was suggested by Nilsson [8]:

$$\hbar \omega_0 = 41A^{1/3}.$$

The pairing correction ( $\delta P_Z$  and  $\delta P_N$ , corresponding to proton and neutron numbers, respectively) is calculated using the BCS method [9]. Then, the relation for the total energy of the nucleus ( $Z, N$ ) results:

$$DE(Z, N, \varepsilon) = \tilde{U}(Z, N, \varepsilon) - \tilde{U}_s(Z, N) + \delta U_N(Z, N, \varepsilon) + \delta U_Z(Z, N, \varepsilon) + \delta P_N(Z, N, \varepsilon) + \delta P_Z(Z, N, \varepsilon) \quad (11)$$

where  $\tilde{U}(Z, N, \varepsilon)$  is the energy of a nucleus ( $Z, N$ ) having deformation  $\varepsilon$ , and  $\tilde{U}_s(Z, N)$  the energy in its spherical shape.

The shape of the nucleus is an ellipsoid. The deformation parameter  $\varepsilon$  is defined as:

$$\varepsilon = \frac{\Delta R}{R_0},$$

where  $\Delta R$  is the difference between the major and the minor axis and  $R_0$  is the average nuclear radius.

As one said, the total fragments kinetic energy is close to the available energy for light and heavy complementary fragments with masses around  $A = 104$  and  $A = 132$ , respectively. Let us relate this result to the deformation for nuclei in this mass neighborhood.

The energies of nuclei  $^{106-108}\text{Mo}$  and  $^{106-108}\text{Tc}$  as a function of their corresponding deformations ( $\varepsilon$ ) are presented on Figs. 1 and 2, respectively. The assumed Nilsson's constants for these nuclei are

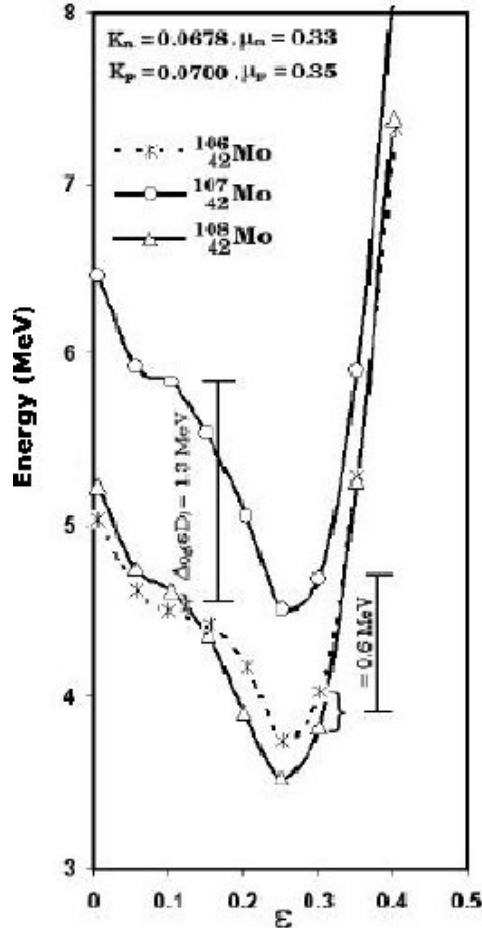
$$\kappa_N = 0.0678, \kappa_p = 0.07, \\ \mu_N = 0.33 \text{ and } \mu_p = 0.35.$$

As we can see, those nuclei have a prolate shape with to  $\varepsilon = 0.3$  in their ground state. If the fragment deformation changes from  $\varepsilon = 0$  to  $\varepsilon = 0.3$  the deformation energy will decrease by around 2 MeV, while a change from  $\varepsilon = 0.3$  to  $\varepsilon = 0.4$  increases of deformation energy by 4 MeV. This result suggests that these nuclei are prolate and soft

between  $\varepsilon = 0$  to  $\varepsilon = 0.3$  and became stiff for higher prolate deformations.

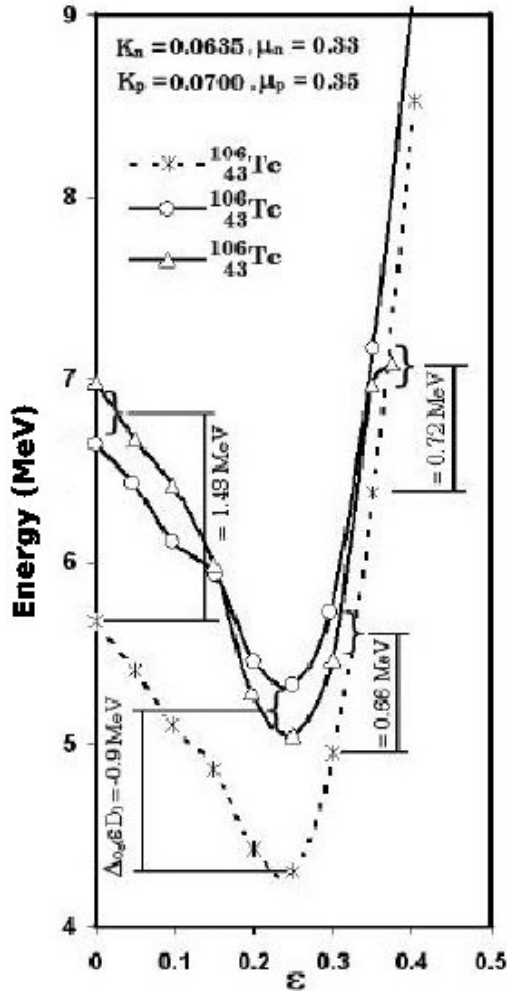
The energy as a function of deformation for nuclei  $^{130-132}\text{Sn}$  are presented on Fig.3. The assumed Nilsson's constants for these nuclei are:

$$\kappa_N = 0.0635, \kappa_p = 0.067, \\ \mu_N = 0.43 \text{ and } \mu_p = 0.54.$$



**Figure 1.** Deformation energy for nuclei  $^{106-108}\text{Mo}$  calculated by a drop liquid model with pairing and shell correction [6]. See text.

One can see that  $^{130}\text{Sn}$  is softer than  $^{132}\text{Sn}$ . For a deformation from  $\varepsilon = 0$  to  $\varepsilon = 0.2$ , the nucleus  $^{130}\text{Sn}$  spends around 5 MeV while the nucleus  $^{132}\text{Sn}$ , for the same deformation, spends 10 MeV. The neutron number  $N = 82$  and proton numbers around  $Z = 50$  correspond to spherical hard nuclei.



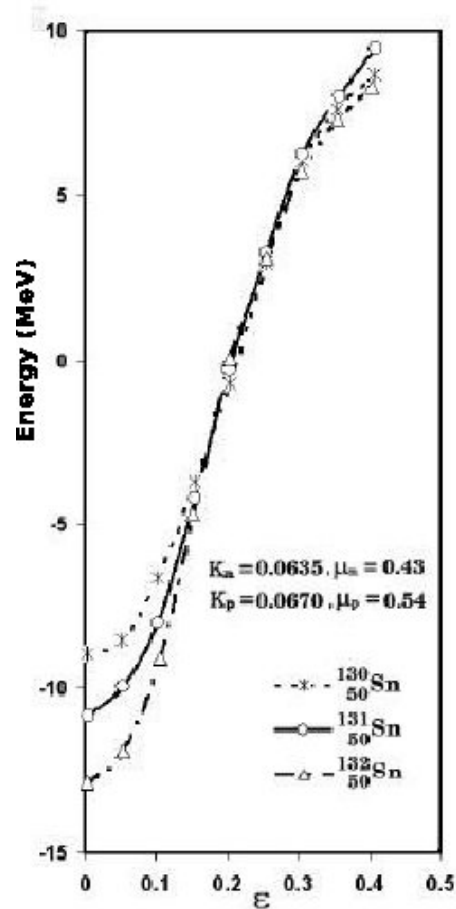
**Figure 2.** Deformation energy for nuclei  $^{106-108}\text{Tc}$  calculated by a drop liquid model with pairing and shell correction [6].

The above characteristics of light fragments, corresponding to masses from  $A=100$  to  $A=106$ , and their complementary fragments, corresponding to mass from  $A=130$  to  $A=132$ , makes possible that their maximal values of the total kinetic energy of complementary fragments  $TKE$  are close to the available energy.

For the case of  $^{233}\text{U}(n_{th}, f)$ , the total kinetic energy of the couple ( $_{42}\text{Mo}_{62}, _{50}\text{Sn}_{80}$ ) is almost equal to the available energy. This result means that the corresponding scission configuration is composed by fragments in their ground state. On the Fig. 4 we can see the several equipotential energy of the scission configuration composed by those fragments given by the relation:

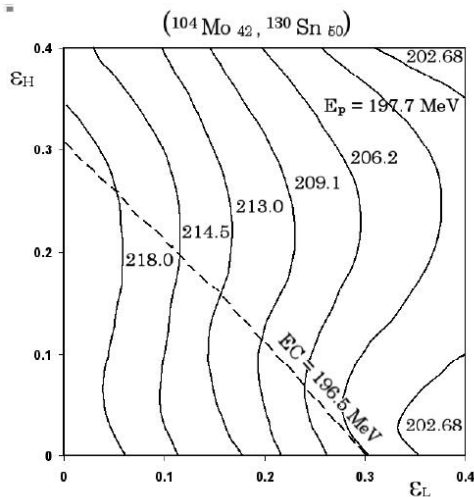
$$V = CE(\varepsilon_H, \varepsilon_L) + DE_H(\varepsilon_H) + DE_L(\varepsilon_L) \quad (13)$$

where  $DE_H$  and  $DE_L$  are the heavy and light fragment deformation energy, respectively, calculated using the Nilsson model [8] and  $CE$  is the Coulomb interaction energy between the two fragments separated by 2 fm. On this curve one obtains that for  $\varepsilon_H = 0$  and  $\varepsilon_L = 0.3$  the Coulomb energy is equal to the available energy to 204 MeV.



**Figure 3.** Deformation energy for nuclei  $^{130-132}\text{Sn}$  calculated by a drop liquid model with pairing and shell correction [3]. See text.

The results are similar to complementary fragments corresponding to the deformed transitional nuclei with  $A_L$  between 100 and 106 ( $N$  between 60 y 64) and to the spherical nuclei with  $A_H$  around 132 ( $Z = 50$  and  $N = 82$ ).



**Figure 4.** Equipotential curves for scission configuration of fragments  ${}_{42}\text{Mo}_{62}$ ,  ${}_{50}\text{Sn}_{80}$  as a function of their deformation.  $\varepsilon_L$  and  $\varepsilon_H$  are the light and heavy fragment deformation with Nilsson parameters [7].

For the complementary fragments  ${}_{42}\text{Mo}_{62}$  and  ${}_{50}\text{Sn}_{80}$ , the maximal value of CE corresponds to ground state nuclei or close to that. This case is unique. Other configurations will need deformation energy, which will be higher for the harder nuclei. On the Fig. 3 is presented the deformation energy for the spherical nuclei  ${}^{130}\text{Se}$ ,  ${}^{131}\text{Se}$  and  ${}^{132}\text{Se}$ , respectively. We can see that the double magic nucleus  ${}^{132}\text{Se}$  need 2 MeV more than  ${}^{130}\text{Se}$  for going from the spherical state  $\varepsilon = 0$  to the slightly deformed  $\varepsilon = 0.05$ . The fact that  ${}^{132}\text{Se}$  is no so hard as  ${}^{132}\text{Se}$  explain why the highest values of Coulomb interaction energy correspond to values close to the available energy for  ${}^{233}\text{U}(n_{th},f)$  as well as for  ${}^{235}\text{U}(n_{th},f)$ .

#### 4. Conclusion

From calculations of scission configurations from thermal neutron induced fission of  ${}^{233}\text{U}$  and  ${}^{235}\text{U}$ , respectively, one can conclude that the highest value of Coulomb interaction energy between complementary fragments corresponds to fragmentations  $({}_{42}\text{Mo}_{62}, {}_{50}\text{Sn}_{80})$  and  $({}_{42}\text{Mo}_{64}, {}_{50}\text{Sn}_{80})$ , respectively. For both cases the calculated maximal values of Coulomb interaction energy values are equal to the available energy of the reaction for spherical ( $\varepsilon_H = 0$ ) heavy fragments and prolate ( $\varepsilon_L = 0.3$ ) complementary light fragments, which correspond to their ground states. Moreover the light fragments are soft

between  $\varepsilon_L = 0$  and  $\varepsilon = 0.3$  and harder if they go to more prolate shapes; while the spherical heavy fragment  ${}_{50}\text{Sn}_{80}$  is no as hard as the fragment  ${}_{50}\text{Sn}_{82}$ . The calculated maximal value of Coulomb interaction energy is equal to the measured maximal value of total kinetic energy of fragments. The pre-scission kinetic energy and intrinsic excitation energy of fragments are assumed to be null. These results suggest that fission process take time to explore all energetically permitted scission configurations.

#### 5. References

- [1]. Möller P, Madland DG, Sierk AJ, Iwamoto A. Nature. 2001; 409:785.
- [2]. Dickmann F, Dietrich K. Nucl. Phys. 1969; A129:241.
- [3]. Wilkins BD, Steinberg EP, Chasman RR. Phys. Rev. 1976; C14:1832.
- [4]. Signarbieux C, Montoya M, Ribrag M, Mazur C, Guet C, Perrin P, Maurel M. J. Phys. Lett. (Paris), 42(1976)L-437 - L-440.
- [5]. Strutinsky VM. Nucl. Phys. 1967; A95: 420.
- [6]. Myers WD, Swiatecky WS., Nucl. Phys. 1966; 81:1.
- [7]. Quentin P, Babinet R. Nucl. Phys.1970; A159: 365-384.
- [8]. Nilsson CG. Mat. Fys. Medd. Dan. Vid. SelsK 29 (1955) n.16.
- [9]. Belyaev ST. Mat. Fys. Medd. Vid. Selsk. 1959; 31(11).