# Neutron emission effects on fragment mass and kinetic energy distribution from fission of $^{239}$Pu induced by thermal neutrons

Modesto Montoya[1,2,*], Justo Rojas[1,3], Iván Lobato[1]

[1] Instituto Peruano de Energía Nuclear, Departamento de Física, Av. Canadá 1470, Lima 41, Perú

[2] Facultad de Ciencias, Universidad Nacional de Ingeniería, Lima, Perú

[3] Facultad de Ciencias Físicas, Universidad Nacional Mayor de San Marcos,
Apartado postal 14-0149, Lima 14, Perú

## Resumen

Tsuchiya *et al.* han medido los valores del promedio de la energía cinética <E*****>, la multiplicidad neutrónica ($\bar{v}$) en función de la masa (m*****), así como el rendimiento de masa (Y (m*****)) de los fragmentos de la fisión inducida por neutrones térmicos del $^{239}$Pu. La masa y la energía cinética han sido calculadas a partir de los valores medidos de energía cinética de un fragmento y la diferencia de tiempos de vuelo de los fragmentos complementarios. Sin embargo, los mencionados autores no presentan sus resultados acerca de la desviación estándar $\sigma_E$***** (m*****). En este trabajo hemos hecho una simulación numérica de ese experimento, suponiendo una distribución inicial de la energía cinética de fragmentos primarios (E(A)) con una desviación estándar constante en función de la masa de los fragmentos ($\sigma_E$ (A)). Como resultado de la simulación de ese experimento, obtenemos la curva $\sigma_E$***** (m*****), la que presenta un ensanchamiento entre m***** = 92, y m***** = 110, así como un pico en m***** = 121.

Kewwords: Monte-Carlo; fisión; $^{239}$Pu; energía cinética de fragmentos; desviación estándar.

## Abstract

The average of fragment kinetic energy (<E*****>) and the multiplicity of prompt neutrons ($\bar{v}$) as a function of fragment mass (m*****), as well as the fragment mass yield (Y (m*)) from thermal neutron induced fission of $^{239}$Pu, have been measured by Tsuchiya *et al*. In that work the mass and kinetic energy are calculated from the measured kinetic energy of one fragment and the diference of time of flight of the two complementary fragments. However they do not present their results about the standard deviation $\sigma_E^*$ (m*****). In this work we have made a numerical simulation of that experiment, assuming an initial distribution of the primary fragment kinetic energy (E(A)) with a constant value of the standard deviation as function of fragment mass ($\sigma_E$ (A)). As a result of that simulation we obtain the dependence $\sigma_E$***** (m*****) which presents an enhancement between m* = 92 and m* = 110, and a peak at m* = 121.

Keywords: Monte-Carlo; fission; $^{239}$Pu; fragment kinetic energy; standard deviation.

## 1. Introduction

The characteristics of the distribution of fission products from thermal-neutron induced fission of heavy nuclei is important for understanding the fission process [1–3]. One of the most important quantities to understand the fission process are the fragment mass and kinetic energy distribution, which is closely related to the topological features in the multi-dimensional potential energy surface [4]. Structures on the distribution of primary (before neutron emission) mass and kinetic energy may be interpreted by shell effects on potential energy of the fissioning system, determined by the Strutinsky prescription and discussed in [5–7]. One expression of the above mentioned primary kinetic energy distribution is constituted by the average value (<E>) and the standard deviation ($\sigma_E$) as a function of primary mass (A). The difficulty is that only final fragments (after neutron emission) are accessible to experimental measurement.

At the Lohengrin mass spectrometer at ILL (Grenoble, France), for the neutron-induced

---

*Corresponding author: mmontoya@ipen.gob.pe fission of $^{235}$U , the final fragment kinetic energy as a function of final mass e(m) distribution was experimentally studied by Brissot *et al.* [8]. Their results show a pronounced broadening of σe around m ≈ 109 and m ≈ 125. A Monte Carlo simulation result suggests that the broadening does not exist on the $\sigma_E$ of the primary fragment kinetic energy expressed as a function of the primary fragment mass. It was shown by means of Monte Carlo simulation made by Montoya *et al*. [8], that the broadening on the $\sigma_e$ curve around the final fragment masses m = 109 and m = 125 can be reproduced without assuming an ad hoc initial structure on $\sigma_E$(A) . Similar experimental results on $\sigma_e$ from $^{233}$U fission induced by neutrons, carried out by Belhafaf *et al.* [9], present a peak around m = 109 and another around m = 122. The authors attribute the first peak to the evaporation of a large number of neutrons around the corresponding mass number, i.e. there is no peak on the standard deviation of the primary kinetic energy distribution ($\sigma_E$) as a function of primary fragment mass (A). The second peak is attributed to a real peak on $\sigma_E$ (A).

However, theoretical calculations related to primary distributions [10] do not suggest the existence of that peak. A Monte Carlo simulation [8] reproduces a pronounced peak on σe (m) curve around m = 109, a depletion from m = 121 to m = 129, and conclude that there is no peaks on $\sigma_E$ (A) curve and the observed peaks on σe (m) are due to the emitted neutron multiplicity and the variation of the average fragment kinetic energy as a function of primary fragment mass.

More recently, Tsuchiya *et al.* [11] have measured simultaneously fragments and prompt neutrons for fission of $^{239}$Pu induced by thermal neutrons from the Kyoto University Reactor (KUR). For each fission event they measure a fragment energy, the difference of time of flight of complementary fragments and the energy of emitted neutrons. Among other results they present: i) the neutron multiplicity vs. the fragment mass ii) the average neutron energy vs. the fragment mass < η > (m***** ), iii) the average of total neutron multiplicity vs. the fragment total kinetic energy, $\nu^{tot}$ <TKE>, iv) the slope of the neutron multiplicity as a function of fragment total kinetic energy, −d < ν > /d < T KE > vs. the fragment mass and v) the average fragment kinetic energy, <E*****> , vs. fragment mass m***** . However they don't present the standard deviation of the kinetic energy distribution as a function of fragment mass $\sigma_E^*$ (m*****).

The aim of the present work is to study the perturbation on mass and kinetic energy of fragments, created by neutron emission, which produces a standard deviation (SD) of the distribution of final fragment kinetic energy ($\sigma_E^*$(m*****)) different from the orresponding to the initial one ($\sigma_E$ (A)). Using the experimental data obtained by Tsuchiya *et al.*[11] and assuming a constant SD of primary kinetic energy distribution ($\sigma_E$ (A)) as input, we made a Monte Carlo simulation of the their experiment; and we calculate the SD of E***** distribution as a function of m***** .

## 2. Monte Carlo simulation model

In our Monte Carlo simulation the input quantities are the primary fragment yield (Y), the average kinetic energy ( $\bar{E}$ ), the SD of the kinetic energy distribution ($\sigma_E$) and the average number of emitted neutron $\bar{\nu}$ as a function of primary fragment mass (A). The output of the simulation are the mass yield *ν* (Y (m*****)), the SD of the kinetic energy distribution ($\sigma_E$ $^*$ ) and the average number of emitted neutron ( $\bar{\nu}$ ) as a function of fragment mass(m***** ). Our goal is to clarify if a $\sigma_E$ (A) with a constant value as input may produce a $\sigma_E$ $^*$ (m*****) curve with structures. In order to simplify the calculation we assume that i) the E values obeys a Gaussian distribution, ii) the average number of emitted neutrons $\bar{\nu}$ corresponds to the fragments with the average value of kinetic energy <TKE> and iii) for emitted neutron number we take the integer part of:

$$N = 0.5 + \bar{\nu}\left[1 - \beta\left(\frac{TKE - \overline{TKE}}{\sigma_{TKE}}\right)\right], \quad (1)$$

where β values are related to the additional excitation energy necessary to emit one more neutron. These values are taken from the extrapolation of results obtained by Tsuchiya *et al.* [11].

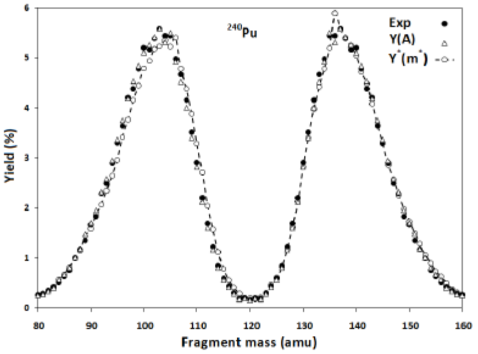

**Figure 1.** Simulation results for the primary (Δ ) and provisional (◦) mass yields, from fission of neutrons, are presented together with experimental data (•), taken from Ref. [11]. $^{239}$Pu induced by thermal neutrons.

### *2.1 Simulation process*

We make several iterative simulations. For the first simulation we chose the primary values Y (A), E(A) and ν (A) equal to the corresponding measured quantities Y (m*****), E* (m***** ) and ( $\bar{\nu}$ (m*****)) (Ref. [11]). The SD of primary fragment kinetic energy distribution as a function of mass, $\sigma_E$ , is taken equal to 5 MeV. Comparing the final simulated quantities to the experimental results, we get a shift that is corrected with a new ensemble of primary quantities. We repeat this process several times until we get final simulated quantities reasonably close to the measured quantities.

In the simulation, for each primary mass A, the kinetic energy of the fission fragments is chosen randomly from a Gaussian distribution,

$$P(E) = \frac{1}{\sqrt{2\pi}\sigma_E} exp\left[-\frac{(E-\overline{E})^2}{2\sigma_E^2}\right], \quad (2)$$

where P (E) is the probability density of energy with average value E and SD $\sigma_E$ . Furthermore, assuming that the fragments loose energy only by neutron evaporation and not by gamma emission or any other process, recoil effect is negligible, and the final fragment kinetic energy is equal to the measured value (kinetic energy E***** ), we get the relation,

$$E^* = (1 - \frac{N}{A})E, \quad (3)$$

which is one of the quantities measured for each fission event by Tsushiya *et al.* [11] As mentioned above, another measured quantity is the time of flight ($t_c$) of the complementary fragment whose distance of flight is $L_c$ . Then, the fragment velocity is $v_c = L_c / t_c$ . Having the fragment kinetic energy E***** (whose corresponding mass is identified by m*****) and the velocity of its complementary fragment $v_c$ (whose mass is identified by $m^*_c$), assuming that linear momentum and mass conservation relations still valid, i.e.

$$m_c^* v_c = \sqrt{2m^* E^*} \quad (4)$$

$$m_c^* + m^* = M_0, \quad (5)$$

$$m^{*2} - 2(M_0 + \frac{E^*}{v_c{}^2})m^* + M_0{}^2 = 0. \quad (6)$$

and

where $M_0$ is the fissioning nucleus mass, we obtain a quadratic equation for m*****

from this equation we obtain

$$m^* = M_0 + \frac{E^*}{v_c^2}\left[1 - \sqrt{\frac{2M_0 v_c^2}{E^*} + 1}\right]. \quad (7)$$

The other solution (m***** > $M_0$ ) does not agree with mass conservation condition.

Let *v* and *L* be the velocity and the length of flight, respectively, of fragments whose kinetic energy is E***** . If the measured difference of time of flight of complementary fragments (Δt) is taken, using the relation

$$\Delta T = \frac{L_c}{v_c} - \frac{L}{v}, \quad (8)$$

and the relations (4) and (5), we obtain

$$m^* = B' - \sqrt{B'^2 - C'^2}, \quad (9)$$

where

$$B' = (\frac{\Delta T}{L + L_c})^2 E^* + C', \text{and} \quad C' = \frac{L_c}{L_c + L} M_0$$

From relations (7) and (5) we calculate $m^*_c$ , and finally, the kinetic energy of complementary mass

$$E_c^* = \frac{1}{2} m_c^* v_c{}^2. \quad (10)$$

With the ensemble of values corresponding to m* , E* and N, we calculate Y (m*), E* (m*), $\sigma_E^*$ (m*) and ν(m*), and the values corresponding to complementary fragments.

In order to obtain an acceptable statistics during the simulation, we have considered a total number of fission events of $^{239}$Pu of the order of $10^8$, and we have computed the standard deviation of all the relevant quantities by means of the following expression:

$$\sigma^2(m^*) = \frac{\sum_{j=1}^{N_j(m^*)} E^{*2}_j(m^*)}{N_j(m^*)} - \bar{E}^{*2}(m^*),$$

where E* (m*), is the average value of the kinetic energy of final fragments with a given mass m* , and $N_j$ (m*) is the number of fission events corresponding to that mass.

## 3. Results and interpretation

The simulated provisional mass yield Y (m*) and the primary mass yield Y(A) are illustrated in Fig. 1. For fragments with *m* < 103, Y (m*) is lower than Y (A) while for m > 103, Y (m*) is higher than Y (A).

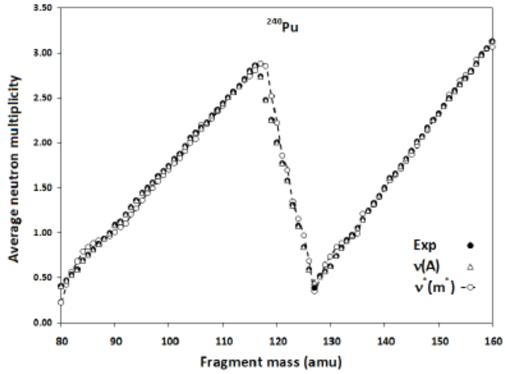

**Figure 2.** The average number of emitted neutrons from fission of from Ref. [11]. $^{239}$ Pu induced by thermal neutrons: as a function of the primary ( Δ ) and provisional (◦) fragment mass, both as result of simulation, and experimental extrapolated values (•).

The simulated average number of emitted neutron ν (m* ) is approximately equal to ν (A). See Fig. 2. The plots of the simulated average kinetic energy for the primary and final fragments as function of their corresponding masses are shown in Fig. 3. In general, the simulated average final kinetic energy as a function of measured mass (E* (m*)) is lower than E(A). The standard deviation of the kinetic energy of fission fragments $\sigma_E^*$ , calculated from the results of the simulation within the framework of the proposed model is shown in Fig.4. We observe an enhancement of $\sigma_E^*$ between m* = 92 and m* = 110. As we can see one of the main features is the presence of a sharp peak at m* ≈ 121, which is due to the 3 MeV jump of primary fragment kinetic energy from A = 121 to A = 122, see Fig. 3. Actually, we are studying only abrupt parameters variation effects on SD of E* -values distribution; it is not necessary to take into account other similar experimental results on Y (m*), ν(m*) and E* (m*).

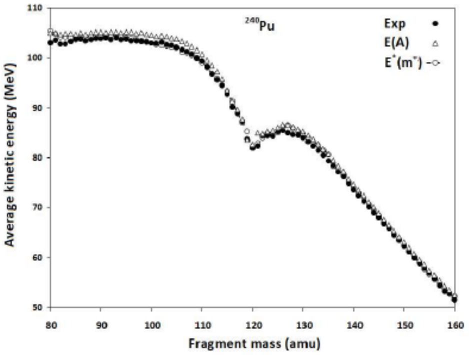

**Figure 3.** Average kinetic energy of as a function of primary(Δ) and provisional (◦) fragment mass, respectively, from fission induced of $^{239}$Pu by thermal neutrons, as a result of simulation in this work, to be compared to experimental data (• ) from Ref. [11].

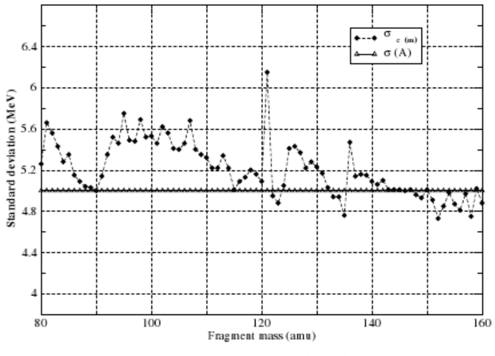

**Figure 4.** SD of fragment kinetic energy as a function of primary (Δ) and provisional (◦) fragment mass, respectively, from fission of $^{239}$Pu induced by thermal neutrons, as a result of simulation in this work. Tsuchiya *et al.* did not presented their corresponding experimental data that may be compared with this results.

## 4. Conclusions

As a result of the simulation of an experiment measuring the average of fragment kinetic energy (<E**>*) and the multiplicity of prompt neutrons <ν> as a function of fragment mass (m***** ), as well as the fragment mass yield (Y (m***** )) from thermal neutron-induced fission of $^{239}$Pu made by Tsuchiya *et al.* [11] we obtain the curve $\sigma_E^*$ (m ) which presents an enhancement between m = 92 and m = 110, and a peak at m***** = 121. It would be very useful to have the experimental result on the curve E*, measured by Tsuchiya *et al.* [11] which may be compared with the result of our simulation of their experiment.

## 5. References

[1] Kornilov NV, Hambsch FJ, A.S. Vorobyev AS. Neutron emission in fission. Nuclear Physics A. 2007; 789 (1-4):55-72.

[2] Quade U, Rudolph K, Skorka S, Armbruster P, Clerc HG, Lang W, Mutterer M, Schmitt C, Theobald JP, Gnnenwein F, Pannicke J, Schrader H, Siegert G, Engelhardt D. Nuclide yields of light fission products from thermal-neutron induced fission of $^{233}$U at different kinetic energies. Nuclear Physics A. 1988; 487(1):1-36.

[3] Wagemans C, Allaert E, Caitucoli F, D'hondt P, Barreau G, Perrin P. Experimental study of some important characteristics of the thermal neutron induced fission of $^{237}$Np. Nuclear Physics A. 1981; 369(1):1-14.

[4] Möller P**,** Madland DG, Sierk AJ, Iwamoto A. Nuclear fission modes and fragment mass asymmetries in a five-dimensional deformation space. Nature. 2001; 409:785-790.

[5] Dickmann F, Dietrich K. On a simple model explanation of shell effects in nuclear fission. Nuclear Physics A. 1969; 129(2):241-258.

[6] Wilkins BD, Steinberg EP, Chasman RR. Scission-point model of nuclear fission based on deformed-shell effects. Physical Review C. 1976; 14:1832-1836.

[7] Fan Tie-shuan, Hu Ji-min, Bao Shang-lian. Study of multichannel theory for the neutron induced fissions of actinide nuclei. Nuclear Physics A. 1995; 591(2):161-181.

[8] Brissot R, Bocquet JP, Ristori C, Granon J, Guet C, Nifenecker H, Montoya M. Kinetic-energy distribution for symmetric fission of $^{236}$U. In: International Atomic Energy Agency. Physics and Chemistry of Fission 1979. Vol II. Vienna: IAEA, 1980. p. 99-110.

[9] Belhafaf D, Bocquet JP, Brissot R, Ristori Ch, Crancon J, Nifenecker H, Mougey J, Ramamurthy VS. Kinetic energy distributions around symmetric thermal fission of U$^{234}$ and U$^{236}$. Z. Physik A - Atoms and Nuclei.1983; 309(3):253-259.

[10] Faust HR, Bao Z. Higher moments in the kinetic energy distribution of fission products. Nuclear Physics A. 2004; 736(1-2):55-77.

[11] Tsuchiya C, Kakagome Y, Yamana H, Moriyama H, Nishio K, Kanno I, Shin K, Kimura I. Simultaneous measurement of prompt neutrons and fission fragments for $^{239}$Pu($n_{th},f$). Journal Nuclear Science and Technology. 2000; 37(11):941-48.

[12] K. Nishio,M. Nakashima, I. Kimura y Y. Nakagome. Multi-parametric measurement of prompt neutrons and fission fragments for $^{233}$U($n_{th},f$). Journal Nuclear Science and Technology. 1998; 35(9):631-42.